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MINISTRY OF HOME SECURITY.

CIVIL DEFENCE RESEARCH COMMITTEE.

The detonation velocity of solid explosives contained in long cylindrical tubes of varying weight,

by Dr. H. Jones.

Introduction.

The Chapman-Jouguet theory of the detonation of gases contained in rigid tubes has been applied by many authors to the detonation of solid explosives. Calculations by A. Schmidt, R. Becker, Langweiler, and more recently Kistiakowsky and Wilson, have used the theory in an inverted sense to calculate from the observed detonation velocities the covolumes of the gases formed in the detonation process. In the earlier calculations of A. Schmidt and Langweiler the covolumes so determined seemed to bear little relation to the actual size of the molecules. Kistiakowsky and Wilson, however, using a more flexible form for the equation of state, obtain values which, reduced to room temperature, are not incompatible with the b of van der Waal's equation as determined at ordinary pressures. A practical aim of these calculations is the prediction of the detonation velocity of possible new explosives. Applied to explosives which detonate with the development of very high temperatures these calculations, regarded from this practical standpoint, have considerable success; they fail completely when applied to ammonium nitrate or similar explosives, always predicting (with the covolumes determined from other explosives) a velocity far in excess of the observed value. The same failure occurs with calculations made along the lines of R.C.166 (cf. § 5e of this note.)

It seems, at first sight, remarkable that the Chapman-Jouguet theory in no way involves the velocity of the reactions which lead to the breakdown of the explosive molecule and the formation of the product gases. This is because the theory visualizes a steady state in a rigid tube and connects the properties of the unexploded gas with those of the completely reacted gases. The distance which separates these two conditions is immaterial to the theory. There thus appears to be a fundamental distinction between the process of detonation in gases and in solids, viz. that in gases the confining tube remains intact whereas for solid high explosives, however strong the confinement, some expansion, if not complete shattering, always occurs. It is clear, therefore, that for solids the distance, in the steady state, which separates the onset and the completion of the reaction must play a part in determining the detonation velocity. It has been shown by Boyes that, assuming rigid confinement, the pressure is actually higher in the reaction zone than at the point where the reaction is complete. Hence, if the length of the reaction zone is comparable with the diameter of the charge, an appreciable expansion must have occurred before the whole of the explosive is converted. In other words, part of the explosive is being transformed at an effectively lower loading density and therefore the velocity may be expected to be less than that given by the Chapman-Jouguet theory assuming infinitely strong confinement. The fact that for the more powerful high explosives the calculations already mentioned are not unsuccessful must be due to the very high rate of the reactions taking place, so that in general the length of the reaction zone is very small compared with the diameter of the charge. Nevertheless reasons are given in § 5 for believing that in T.N.T. the observed velocities may be as much as 10 per cent. less than the theoretical values assuming rigid confinement.

In a recent communication (R.C.193) Professor G.I. Taylor has analysed the process of an explosion in a long cylindrical tube and has shown how the radius of the tube increases under the adiabatic expansion of the product gases of the detonation wave. The analysis of the present note is along the lines of Professor Taylor's paper; the main difference, in the mathematics, is that in Professor Taylor's calculation there existed a known relation between the pressure and the density, viz. the adiabatic relation, whereas in the present calculations such an algebraic relation does not initially exist but is replaced by a differential relation. In other words, in the present calculations, the differential equation expressing the adiabatic condition cannot be integrated independently but only simultaneously with the equation of motion.

§ 1. The general equations of the detonation process.

We consider the detonation of a long cylindrical charge surrounded by a casing of weight λ per unit length. When the detonation process is fully developed a steady state is established. If the detonation velocity is U we consider the process from axes moving with this velocity. The shock wave front in the solid is then at rest. This point is taken as the origin $x = 0$. Behind the shock wave front the velocity of the reacting material we denote by q which is, of course, a function of x. There is a discontinuous change of pressure and density at the shock wave front. Let $E_u(p, \rho)$ be the energy per unit mass of the unexploded material at pressure p and density ρ reckoned from a zero at $p = 0$. The equations connecting the conditions on each side of the discontinuity are then

$$\left\{ \begin{aligned} \rho_o q_o &= U\Delta \\ p_o &= U\Delta(U - q_o) = U^2\Delta\left(1 - \frac{\Delta}{\rho_o}\right) \\ E_u(p_o, \rho_o) &= \tfrac{1}{2}p_o\left(\frac{1}{\Delta} - \frac{1}{\rho_o}\right) = \tfrac{1}{2}U^2\left(1 - \frac{\Delta}{\rho_o}\right)^2 \end{aligned} \right. \qquad \ldots\ldots (1)$$

where Δ is the unexploded density, and ρ_o, p_o, q_o are the density, pressure, and velocity in the shock wave front.

The equation of continuity, when the angle of the expanding tube is small, can be expressed

$$R^2 q \rho = R_o^2 U \Delta \qquad \ldots\ldots (2)$$

when R_o is the initial tube radius and R the radius at the point x. The equation of motion to the same approximation is

$$q\frac{\partial q}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0 \qquad \ldots\ldots (3)$$

Let $E_e(p, \rho)$ denote the internal energy per unit mass of the converted explosive material, and let n denote the fraction of the explosive which has been converted at a particular point x. Then if E be the total energy per unit mass

$$E = nE_e + (1 - n)E_u \qquad \ldots\ldots (4)$$

The pressure in both parts of E must be the same but the density may, of course, be different.

Also we denote by H_o the whole of the chemical energy released per unit mass when the reaction is complete. At any point x the energy released is denoted by H and therefore

$$H = nH_o \qquad \ldots\ldots (5)$$

In addition to the kinematical equation of continuity (2) and the dynamical equation (3) we have also a thermodynamical equation expressing the conservation of energy. We write this in the form

$$\frac{\partial}{\partial x}(E - H) + p\frac{\partial}{\partial x}\left(\frac{1}{\rho}\right) = 0 \qquad \ldots\ldots (6)$$

This equation assumes that no heat is transferred from one element to the next during the development of the reaction from the shock wave front to the point at which the reaction is complete. We are therefore omitting any consideration of the actual kinetics of the process by which the reaction is propagated,

with the result that the problem of the propagation of detonation is insoluble without a further basic assumption. In the case of gases the assumption generally known as the Chapman-Jouguet condition has proved successful and has been justified theoretically by Boyer for the case in which the mechanism of the reaction depends on heat conduction.

In the present calculations, which attempt to take account of the finite time required for the completion of the reaction in the detonation wave and the consequent expansion during this reaction, we apply the Chapman-Jouguet condition in a slightly generalised form. The assumption made is that at the point where the reaction is complete the velocity is equal to the local velocity of sound. If we denote by X the value of x at the point where the reaction is complete, and the conditions at this point by the subscript $_1$ the above assumption is expressed by

$$q_1 = C_1 \qquad \ldots\ldots (7)$$

where C_1 is the velocity of sound at $x = X$. For infinitely strong confinement this is identical with the usual condition applied to gases and solids. Since

$$C^2 = \left(\frac{dp}{d\rho}\right)_{ad} \qquad \ldots\ldots (8)$$

the condition (7) becomes

$$q_1^2 = \left\{\frac{p_1}{\rho_1^2} - \left(\frac{\partial E_e}{\partial \rho}\right)_1\right\} \Big/ \left(\frac{\partial E_e}{\partial p}\right)_1 \qquad \ldots\ldots (9)$$

In addition to equation (3) there is another dynamical equation determining the expansion of the casing, viz.

$$\frac{\partial^2 R}{\partial x^2} = \left(\frac{2\pi}{\lambda U^2}\right) pR \qquad \ldots\ldots (10)$$

in which t has been replaced by x/U since we are dealing with a steady state.

§ 2. Reduction of the equations to a form suitable for integration.

In order to solve the above equations we make a number of assumptions. These are of a different kind from (7); they express the initial chemical information upon which any theory of detonation must be based, and could be modified without affecting the main conclusions.

First we assume that the reaction velocity is constant throughout the reaction zone, i.e.

$$n = (x/X) \qquad \ldots\ldots (11)$$

Secondly we take a very simple form for E_e, viz.

$$E_e = c\frac{p}{\rho} \qquad \ldots\ldots (12)$$

where c is a constant of the nature of a specific heat. Thirdly we assume that H_o is independent of p and ρ (this has been shown to be very nearly the case for T.N.T.)

$$\therefore \qquad H = \frac{x}{X} H_o \qquad \ldots\ldots (13)$$

Fourthly we assume that in the expression for the energy

$$E = \frac{x}{X} c \frac{p}{\rho} + \left(1 - \frac{x}{X}\right) E_u \qquad \ldots\ldots (14)$$

E_u can be regarded as a constant.

It is convenient to use the following abbreviations:-

$$y = \frac{\Delta}{\rho}, \quad h_o = \frac{H_o}{U^2} \quad \text{and} \quad r = R/R_o.$$

Equation (6) can then be written, making use of (1),

$$c \frac{\partial}{\partial x}\left(\frac{xyp}{U^2\Delta}\right) + X\left(\frac{p}{U^2\Delta}\right)\frac{\partial y}{\partial x} = h_o + \tfrac{1}{2}(1 - y_o)^2 \qquad \ldots\ldots (15)$$

Equation (3) with (2) becomes

$$y \frac{\partial}{\partial x}\left(\frac{p}{U^2\Delta}\right) + \tfrac{1}{2}\frac{\partial}{\partial x}(r^{-4}y^2) = 0 \qquad \ldots\ldots (16)$$

which can be rewritten

$$\frac{\partial}{\partial x}\left(\frac{p}{U^2\Delta}\right) + \frac{\partial y}{\partial x} = \tfrac{1}{2}(1 - r^{-4})\frac{\partial y}{\partial x} + \tfrac{1}{2}\frac{\partial}{\partial x} \cdot y(1 - r^{-4}) \qquad \ldots\ldots (17)$$

and integrating from 0 to x and using equations (1) we have

$$\frac{p}{U^2\Delta} = 1 - y + \tfrac{1}{2}y(1 - r^{-4}) + \tfrac{1}{2}\phi \qquad \ldots\ldots (18)$$

where $$\phi = \int_o^x (1 - r^{-4})\frac{\partial y}{\partial x}\,dx \qquad \ldots\ldots (19)$$

It is convenient to write the equations in this way since $1 - r^{-4}$ is small for heavy confinement, vanishing of course for rigid tubes. If we integrate in a similar way equation (15) we obtain

$$cxy\left\{1 - y + \tfrac{1}{2}y(1-r^{-4}) + \tfrac{1}{2}\phi\right\} + X\left\{y - \tfrac{1}{2}y^2 - y_o + \tfrac{1}{2}y_o^2\right\} \qquad \ldots\ldots (20)$$

$$+ \frac{X}{2}\int_o^x (1-r^{-4})y\frac{\partial y}{\partial x}\,dx + \frac{X}{2}\int_o^x \phi\frac{\partial y}{\partial x}\,dx = \left\{h_o + \tfrac{1}{2}(1-y_o)^2\right\}x$$

A partial integration gives

$$\int_o^x \phi\frac{\partial y}{\partial x}\,dx = \phi y - \int_o^x y(1-r^{-4})\frac{\partial y}{\partial x}\,dx$$

and hence (20) becomes

$$y^2\left\{(X + 2cx) - cx(1-r^{-4})\right\} - (2 + \phi)(cx + X)\,y \qquad \ldots\ldots (21)$$

$$+ \left\{(X + 2h_o x) - (X - x)(1 - y_o)^2\right\} = 0$$

This equation is not merely a quadratic in y since ϕ also depends on y. It is, however, a useful form for our purpose since ϕ is small for heavy confinement, i.e. large λ .

From equation (8) for the velocity of sound we find

$$\left(\frac{c}{U}\right)^2 = \frac{p}{U^2\Delta}\left(\frac{1+c}{c}\right) y \qquad \ldots\ldots (22)$$

and thus the Chapman-Jouguet condition (9) with equation (2) gives

$$\left(\frac{c_1}{U}\right)^2 = \left(\frac{q_1}{U}\right)^2 = y_1^2 r_1^{-4} = \frac{p_1}{U^2\Delta}\left(\frac{1+c}{c}\right) y_1 \qquad \ldots\ldots (23)$$

which can be rewritten with the help of (18)

$$y_1\left\{(1+2c) - c(1-r^{-4})\right\} = (1+c)\left\{1 + \tfrac{1}{2}y_1(1-r_1^{-4}) + \tfrac{1}{2}\phi_1\right\} \qquad \ldots\ldots (24)$$

when all quantities refer to the point where the reaction is complete. Also at $x = X$ (21) becomes

$$y_1^2\left\{(1+2c) - c(1-r^{-4})\right\} - (2+\phi)(1+c)y_1 + (1+2h_o) = 0 \qquad \ldots\ldots (25)$$

which with (24) gives

$$2h_o = \frac{2H_o}{U^2} = \frac{c^2 + (1+c)^2\left[\phi_1 + \frac{1}{4}\phi_1^2 - \frac{1}{4}y_1^2(1-r_1^{-4})^2\right] + c(1-r_1^{-4})}{(1+2c) - c(1-r_1^{-4})} \qquad (26)$$

For infinitely strong confinement $\phi = (1-r^{-4}) = 0$ and we have

$$2H_o = U_\infty^2\left(\frac{c^2}{1+2c}\right) \qquad \ldots\ldots (27)$$

where U_∞ denotes the detonation velocity in this limiting case.

We can write equation (26) therefore as follows

$$\left(\frac{U_\infty}{U}\right)^2 = \frac{1 + \left(\frac{1+c}{c}\right)^2\left[\phi_1 + \frac{1}{4}\phi_1^2 - \frac{1}{4}y_1^2(1-r_1^{-4})^2\right] + \frac{1}{c}(1-r_1^{-4})}{1 - \left(\frac{c}{1+2c}\right)\left(1-r_1^{-4}\right)} \qquad (28)$$

This equation does not, of course, give U until y and r have been obtained as functions of x. This is achieved by successive approximation for the case of heavy confinement.

§ 3. The solution for a non-expanding tube.

In this special case $\phi = 1-r^{-4} = 0$ and $2h_o = c^2/(1+2c)$ and therefore (21) reduces to

$$y^2\left(1 + 2c\,\frac{x}{X}\right) - 2\left(c\,\frac{x}{X} + 1\right)y + \left\{1 + \left(\frac{c^2}{1+2c}\right)\frac{x}{X} - \left(1-\frac{x}{X}\right)(1-y_o)^2\right\} = 0 \qquad (29)$$

We denote the solution of this equation by $y^{(o)}(x)$ and regard it as the zero order approximation for the solution in the case of an expanding tube. The solution of (29) which satisfies the initial condition that $y = y_o$ for $x = 0$ is

$$y^{(o)}(x) = \frac{1 + c\frac{x}{X} - \left\{\left(1+c\frac{x}{X}\right)^2 - \left(2c\frac{x}{X}+1\right)\left(\frac{c^2}{2c+1}\frac{x}{X}+1-\left(1-\frac{x}{X}\right)(1-y_o)^2\right)\right\}^{\frac{1}{2}}}{2c\frac{x}{X}+1} \qquad (30)$$

When $x = X$ we have $y_1^{(o)} = (1+c)/(2c+1)$ which is independent of y_o. In order to see the nature of the variation of $y^{(o)}$ with x we may take a special example $c = \frac{1}{2}$ which corresponds to $y_1^{(o)} = \frac{3}{4}$ or $(p_1^{(o)}/\Delta) = 1.33$. Instead of $y^{(o)}$ we give $1-y^{(o)}$ which is equal to $p^{(o)}/U^2\Delta$.

(Δ/ρ_o) \ (x/X)	0	.2	.4	.6	.8	1.0
.75	.25	.25	.25	.25	.25	.25
.60	.40	.388	.374	.356	.330	.25
.50	.50	.474	.446	.413	.369	.25
.40	.60	.563	.516	.467	.406	.25

It will be seen that the pressure is always greater in the reaction zone than at the point where the reaction is complete. There is no solution possible when $(\Delta/\rho_o) > (1+c)/(1+2c)$, that is it is necessary that $\rho_o \geq \rho_1$ in order that a solution may exist. When $\rho_o = \rho_1$ the density and the pressure are constant throughout the reaction zone.

The rate of change of $y^{(o)}$ with x at the point $x = 0$ is given by the following equation obtained from (30)

$$X\left(\frac{\partial y^{(o)}}{\partial x}\right)_o = \frac{(2c+1)\left(\frac{1+c}{2c+1} - y_o\right)^2}{2(1-y_o)} \qquad \ldots\ldots (31)$$

In the following we denote $X\left(\frac{\partial y^{(o)}}{\partial x}\right)_o$ by a.

§ 4. An approximate solution for an expanding tube when the expansion is small.

We obtain an approximate solution to (21) by calculating $(1-r^{-4})$ and ϕ from the known zero approximation for y, viz. $y^{(o)}(x)$. The first approximation therefore to r as a function of x is given by (10) and (18) neglecting in these equations the terms $(1-r^{-4})$ and ϕ. Hence the equation giving the first approximation to r is

$$\frac{\partial^2 r}{\partial x^2} = \frac{2\pi\Delta}{\lambda}\left(1 - y^{(o)}(x)\right)r \qquad \ldots\ldots (32)$$

We make the further approximation, which the preceding table shows to be not too unsatisfactory, of replacing $y^{(o)}(x)$ by $y_o + a\frac{x}{X}$. The solution of

$$\frac{\partial^2 r}{\partial x^2} = \frac{2\pi\Delta}{\lambda}\left(1 - y_o - \frac{ax}{X}\right)r \qquad \ldots\ldots (33)$$

to first powers of Δ/λ is

$$r = 1 + \frac{\pi\Delta}{\lambda} X^2 \left[\left(1-y_o\right)\left(\frac{x}{X}\right)^2 - \frac{a}{3}\left(\frac{x}{X}\right)^3 \right] \qquad \ldots\ldots (34)$$

and therefore

$$1 - r^{-4} = \frac{4\pi\Delta X^2}{\lambda} \left[\left(1-y_o\right)\left(\frac{x}{X}\right)^2 - \frac{a}{3}\left(\frac{x}{X}\right)^3 \right] \qquad \ldots\ldots (35)$$

and

$$\phi = \frac{4\pi\Delta X^2}{3\lambda} a \left[\left(1-y_o\right)\left(\frac{x}{X}\right)^3 - \frac{a}{4}\left(\frac{x}{X}\right)^4 \right] \qquad \ldots\ldots (36)$$

With equs. (35) and (36) a first approximation to (21) could be obtained. Clearly higher approximations could only be found numerically, i.e. for a particular case in which the numerical values of all the coefficients are known. It would then be possible to find solutions of any degree of accuracy. For the present purpose it is more interesting to obtain general results for the case of heavy confinement, i.e. for small $1-r^{-4}$. To first powers of $\frac{\Delta X^2}{\lambda}$ therefore equation (28) becomes

$$\left(\frac{U_\infty}{U}\right)^2 = 1 + \left(\frac{1+c}{c}\right)^2 \left\{ \phi_1 + \left(\frac{c}{1+2c}\right)\left(1-r_i^{-4}\right) \right\} \qquad \ldots\ldots (37)$$

and using equations (35) and (36) we find

$$\left(\frac{U_\infty}{U}\right)^2 = 1 + \frac{f\pi X^2\Delta}{\lambda} \qquad \ldots\ldots (38)$$

where

$$f = \frac{4}{3}\left(\frac{1+c}{c}\right)^2 \left[\left(1-y_o\right)\left\{ \frac{3c}{1+2c} + a \right\} - a\left\{ \frac{c}{1+2c} + \frac{a}{4} \right\} \right] \qquad \ldots\ldots (39)$$

and a is given by equation (31). Since $(\rho_1^{(o)}/\Delta) = (2c+1)/(c+1)$, the factor f can be expressed in terms of (ρ_o/Δ) and $(\rho_1^{(o)}/\Delta)$. The following table gives an idea of the magnitude of this factor.

$\left(\frac{\rho_o}{\Delta}\right)$ \ $\left(\frac{\rho_1^{(o)}}{\Delta}\right)$	1.2	1.3	1.4	1.5	2.0
1.2	2.79				
1.3	3.86	2.37			
1.4	4.89	2.94	2.04		
1.5	5.86	3.45	2.38	1.78	
2.0	10.35	5.59	3.74	2.73	1.00

If τ denotes the time taken for the whole conversion of the explosive then $X = U\tau$, and if also in place of λ we use the ratio of the weight of the case to the weight of the explosive α so that $\lambda = \pi R_o^2 \Delta\alpha$,

equation (38) may be written

$$\left(\frac{U_\infty}{U}\right)^2 = 1 + \frac{fU^2\tau^2}{\alpha R_o^2} \qquad \ldots\ldots (40)$$

Still another way in which the result can be shown is as follows

$$\frac{U_\infty - U}{U_\infty} = \frac{f}{4}\frac{U^2\tau^2\Delta}{R_o\sigma} \qquad \ldots\ldots (41)$$

where σ is the mass per unit area of the encasing tube. This would be applicable to the case of the variation of U with diameter for tubes of given thickness. The relation between α and R_o shown in (40) has already been noted by Professor Taylor (R.C.193).

§ 5. Comparison with experimental data.

There does not appear to be, unfortunately, much published data in which sufficient details are given to make a quantitative comparison with the foregoing calculations. There are, however, a number of properties of detonating explosives which appear to fit in fairly well with the above equations.

(a) It is observed that although the detonation velocities vary with diameter and degree of confinement the variations are, in the case of the more powerful explosives, only of the order of a few per cent. for comparatively wide variations of α and R_o. It follows, therefore, according to (40), that $X = U\tau$ must be appreciably less than the diameter of the charges ordinarily used, e.g. it must not be more than 1 or 2 mm. for substances such as picric acid or T.N.T.

It is very tempting to correlate the excess width of the well marked trace on moving film photographs with the thickness of the reaction zone. A reason supporting this hypothesis is that the comparative sharpness of the trace makes it very difficult to assign it to temperature radiation since the gases for some distance behind the detonation wave front have still very high temperatures. If not to temperature radiation it would appear to be due to chemi-luminescence, i.e. to the radiation emitted during the breakdown of the explosive molecule and the formation of the gas molecules. P. Laffitte (Ann. de Physique, 1925, 10, p.659) remarks that in all his photographs the trace was quite sharp and that the thickness of the image corresponded closely with the thickness of the tube photographed. That is to say it was not appreciably thicker than would be given by an infinitely thin disc of light of the diameter of the charge. If we accept the hypothesis that the thickness of the disc of light which is photographed in these experiments can be identified with the width of the reaction zone X, Laffitte's experiments show that $X \ll R_o$, which is necessary according to (40) if the detonation velocity is not to be far too dependent on R_o and α .

(b) Such quantitative experimental work as has been published on the variation of detonation velocity with diameter of charge refers mainly to charges with very light paper confinement. The effect of the inertia of the air shock wave is much more difficult to consider than that of a metal casing. However, if we replace σ by an effective surface density σ_{eff} due to the outside gas and the light covering, it makes a very rough comparison with (41) possible.

R. Förg (S.S., 1916, p.37) has given the following figures for the variation of detonation velocity for T.N.T. enclosed in paper covers.

Diameter	Cubic density	Det. velocity	$(U_{max} - U)/U_{max}$
2.0 cms.	0.80 gm/cm^3	3765 m/sec.	.081
3.0 "	0.80 "	3905 "	.047
4.0 "	0.79 "	4054 "	.011
5.0 "	0.83 "	4099 "	.00

In the last column U_{max} denotes the detonation velocity at the greatest diameter used, viz. 5 cms.

If in (41) we take the value of $f/4 \times U^2\tau^2\Delta/\sigma_{eff}$ to be 0.1 cms., the values of $(U_\infty - U)/U_\infty$ would be for the diameters of the above table 0.1, .067, .05, .04 which gives the same sort of variation since $U_{max} < U_\infty$. We have seen, however, that f is of the order of 4 and for the above case $\Delta = .8$ gm/cm^3 and if $U\tau$ is taken to be not more than 1 mm., as photographs suggest, then σ_{eff} is not more than 0.1 gm/cm^2, which is a plausible value being not much more than the weight of a paper covering.

The effect of the variation of the diameter of charge on the detonation velocity is still more pronounced for the less powerful explosives such as ammonium nitrate or potassium nitrate mixtures or some of the 'milder' dynamites. For example, Dautriche found for cheddite an increase of about 20% in the detonation velocity when he increased the diameter from 2 to 4 cms. Also Laffitte finds a detonation velocity for No.1 dynamite about 500 m/sec. less than the value found by Dautriche for the same substance and ascribes the difference to the fact that his tubes were 4 mm. diameter whilst Dautriche used 20 mm. tubes. The increased effect of the diameter or the casing weight for the weaker explosives is in agreement with equation (41), since for these explosives the temperature of the detonation wave is lower than for the powerful high explosives and consequently τ is larger.

(c) Calculations of the detonation velocity of T.N.T. based on Bridgman's compressibility data for nitrogen (R.C.166) were found to be from 12 to 15 per cent. greater than the observed values. These calculations assumed infinitely strong confinement and it appears that part at least of the discrepancy may be due to this assumption. At a loading density of 0.8 gm/cm^3 the above discussion suggests that U might be 10 per cent. less than U_∞ for charges not exceeding 1 cm. diameter and with very light covering.

(d) A characteristic feature of detonation velocity - loading density curves is a tendency to flatten at high loading densities, i.e. for the rate of increase of the detonation velocity with density to become smaller. In the case of some explosives, notably cheddite, the curve actually reaches a maximum beyond which the velocity decreases as density increases. It was shown in R.C.166 that, in general, the temperature of the detonation wave decreased with increasing loading density. Although the reaction mechanism of the detonation wave is not understood at present it is a fair assumption that the rate of reaction will decrease with decreasing temperature; in other words τ will increase as the temperature falls. This could therefore explain the flattening of the U, Δ curve which does not appear to be due to the equation of state, for in none of the cases so far examined has the U_∞, Δ curve shown any such tendency at high densities.

A limitation to the applicability of (38) (apart from the many approximations already discussed) is that λ, in the way it has been used in these calculations, cannot in practice be increased indefinitely. Merely increasing the weight and thickness of the confining tube does not, beyond a certain point, increase λ because of the compressibility and fluidity of all actual substances. Thus the fact that the detonation velocity of T.N.T. at high densities does not appear, judging by published data, to exceed 7,000 m/sec. even with fairly strong confinement does not necessarily mean that this

velocity is the ideal maximum corresponding to infinitely strong tubes, and does not therefore rule out the suggested explanation of the observed diminishing of $(dU/d\Delta)$ for high Δ.

(e) Using the methods of R.C.166 the ideal maximum detonation velocity U_∞ of ammonium nitrate as a function of loading density has been calculated. The results are given below.

Δ gm/cm^3	0.475	0.580	0.708	0.837	0.984
U_∞ m/sec.	2608	3069	3781	4570	5450
$T^\circ K$.	1431	1430	1425	1400	1300

It is interesting that observations of detonation velocities of ammonium nitrate, and of ammonium nitrate with small quantities of other substances, vary very extensively between different observers and appear to be very sensitive to the conditions of confinement, and even more to the addition of small quantities of oxidisable material, e.g. carbon or aluminium. The actual values of U (cf. Marshall, Explosives) are considerably less than the values of U_∞ given above at the corresponding loading density. These facts appear to fit nicely with the very low temperature of the detonation as shown in the above table which indicates an exceptionally large reaction time and therefore, according to (41) an exceptional sensitivity to the conditions of confinement. The addition of carbon or aluminium allows for the absorption of the excess oxygen in the detonation products of ammonium nitrate and the formation of CO_2 or Al_2O_3. The great heats of formation of these molecules result in a considerable increase in the temperature of the detonation wave and therefore to a reduction in the reaction time, which according to (41) brings the detonation velocity closer to U_∞.

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